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Multi-Criteria decision making based on PROMETHEE method

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***Abstract*—Given a finite set of alternatives, the multi-criteria decision making problem consists in ranking each alternative from the best to the worst ones. In this paper, we are interested in multi-criteria ranking problems in the existing method PROMETHEE(Preference Ranking Organization Method for Enrichment Evaluations).This method requires the elicitation of preferential parameters in order to construct a preference model which the decision maker accept as a working hypothesis in the decision aiding study. A direct elicitation of these parameters requiring a high cognitive effort from the decision maker, proposed an interactive aggregation-disaggregation approach that infers the PROMETHEE parameters indirectly from holistic information, i.e., training examples. In this approach, the determination of PROMETHEE parameters that best restore the training examples is formulated through a linear program. In this paper, we consider the subproblem of the determination of the weights only(the thresholds being fixed). The subproblem leads to solve a linear program. Numerical experiments were conducted so as to check the behavior of this disaggregation tool. Results shows that this tool is able to infer weights that restores in a stable way the training examples.**

Keywords-PROMETHEE method; preference information; multi-criteria decision making; linear programming

I. INTRODUCTION

The Multi-Criteria Decision Aid (MCDA) has been one of the very fast growing areas of Operational Research (OR) during the two last decades. Multi-Criteria decision aiding often deals with ranking of many concrete alternatives from the best to the worst ones or selecting the best alternatives based on multiple conflicting criteria. The MCDA is also concerned with theory and methodology that can treat complex problems encountered in management, business, engineering, science, and other areas of human activity.

There are two families of methodology for dealing with MCDA problem. One is based on multi-criteria utility function. The other is the outranking methods. ELECTRE (Elimination Et Choix Traduisant larealite) and PROMETHEE are the representatives of the outranking methods.

The general theory and methods to infer weights for MCDA problem have been illustrated in [4-6]. The sorting method based on ELECTRE III has been given in [7]. Although the MCDA methods using PROMETHEE under the incomplete information have been appeared in [8], indifference threshold and preference threshold are not taken into account. In order to reflect the real environment of MCDA, this paper uses the criterion with linear preference and indifference area of PROMETHEE for MCDA problem, in which the weights of criteria are unknown whereas indifference threshold and preference threshold are given by the decision maker. Through the preference relation of some pairs of alternatives, a linear programming is constructed to infer the weights of criteria which are consist to the preference of decision maker. At last, the alternatives can be ranked from the best to the worst one accord to the net flow using the PROMETHEE II method. With the un-predefined weights of criteria, this method reduces the influence of subjective factor to decision making. Taking account of the indifference and preference threshold, this method avoids the situation, in which small difference on some criterion can induce large difference of evaluations on alternatives. This paper is organized as follows. The first section will describe how the PROMETHEE method works. The method of inferring the weights of criteria is given in section III. In section IV, a numerical example is given to illustrate the method. At last, the conclusion of the paper is given.

II. PROMETHEE METHOD

PROMETHEE method is proposed by Brans and it is one kind of method based on outranking relation between pairs of alternative. The outranking method compares pairs of alternatives on each criterion firstly. The PROMETHEE method induces the preferential function to describe the preference difference between pairs of alternative on each criterion. Thus preference functions about the numerical difference between pairs of alternatives are built to describe the preference deference from the point of the decision maker's view. These functions' value ranges from 0 to 1. The bigger the function's value is, the difference of the preference becomes larger. When the value is zero, there is no preferential difference between pair of alternative. On the contrary, when the value is one, one of the alternatives is strictly outranking the other.

Let $A_1, A_2, \cdots, A_m$ be m alternatives and $g_1, g_2, \cdots, g_n$ be n cardinal criteria and let y_{ij} be the criteria value of the i th alternative A_i with respect to the j th criterion g_j. We will assume, without loss of generality that all criteria are to be maximized.

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We use $p_j(A_i, A_k)$ to denote the preference function on criterion g_j

$$P_j(A_i, A_k) = \begin{cases} 0 & y_{ij} \le y_{kj} \\ P(y_{ij} - y_{kj}) & y_{ij} > y_{kj} \end{cases} \quad (1)$$

In Brans et al, six such functions based on different criterion were introduced. They are true criterion, quasi criterion, criterion with linear preference, level criterion, criterion with linear preference and indifference area, and gauss criterion.

Clearly, different generalized criterion represents different altitude towards preference structure and the intensity of preference. It was observed by Brans that the criterion with linear preference and indifference area has been mostly used by user followed by gauss criterion for practical application. In both criteria, the intensity of preference changes gradually from 0 to 1, while in the other criteria, there are sudden changes in the intensity of preference.

In this paper, criterion with linear preference and indifference area is used. The preference function is defined as follows:

$$P_j(d_j) = \begin{cases} 0 & d_j \le s_j \\ (|d_j| - s_j)/(r_j - s_j) & s_j < d_j \le r_j \\ 1 & d_j > r_j \end{cases} \quad (2)$$

Where $d_j = y_{ij} - y_{kj}$ denotes the preference difference between pair of alternatives on criterion g_j. r_j and s_j are preference and indifference threshold, respectively.

Using the preference function, we can get the preference degree for each pair of alternatives on each criterion. In order to get the overall preference degree $S(A_i, A_k)$, the preference degree on each criterion should be aggregated following the formula:

$$S(A_i, A_k) = \sum_{j=1}^{n} \omega_j P_j(A_i, A_k) \quad (3)$$

Where ω_j represent the weight of the criterion g_j,

In order to rank the alternatives from the best one to the worst one, the outgoing and incoming flow for each alternative is defined as follows:

The outgoing flow of alternative $\boldsymbol{A_i}$ is defined as

$$\phi^+(A_i) = \sum_{A_k \in A} S(A_i, A_k) \quad (4)$$

And the incoming flow of alternative $\boldsymbol{A_i}$ is defined as

$$\phi^-(A_i) = \sum_{A_k \in A} S(A_k, A_i) \quad (5)$$

Based on the outgoing flow and incoming flow, the net flow $\phi(A_i)$ is defined by (6) to represent the overall preference degree of alternative A_i and A_j

$$\phi(A_i) = \phi^+(A_i) - \phi^-(A_i) \quad (6)$$

Alternatives can be ranked from the best to the worst one by the net flow. If $\phi(A_i) = \phi(A_j)$, the alternative A_i is indifferent to A_j. If $\phi(A_i) > \phi(A_j)$, the alternative A_i is preferential to A_j.

III. INFERRING THE WEIGHT OF CRITERION

This paper is based on the hypothesis that the preference and indifference thresholds are known, but the weights of criteria are unknown. Additionally, decision maker can give preference relation of some alternatives. We assume that the preference relations of part of the alternatives set $A^T = \{a_1, a_2, \cdots a_k\} \subset A, (k < m)$ are known. For each pair of the alternative a_i, a_k in A^T, one of the two following relations holds: a_iSa_k, a_kSa_i. a_iSa_k represents that decision maker prefers alternative a_i to. In order to infer the weights of criteria, the following linear programming (7-10) was constructed.

$$\max \quad \sum_{i=1}^{k} \sum_{j=i+1}^{k} e_{ij} \quad (7)$$

s.t $S(A_i, A_j) - e_{ij} \ge S(A_j, A_i)$ for all pairs of alternatives in A^T satisfied the relation of A_iSA_j (8)

$S(A_i, A_j) + e_{ij} \ge S(A_j, A_i)$ for all pairs of alternatives in $\boldsymbol{A^T}$ satisfied the relation of A_jSA_i (9)

$$\boldsymbol{e_{ij} \ge 0} \quad (10)$$

In the above linear programming, there are $k(k-1)/2$ nonnegative decision variables. The goal function is a linear function about the unknown weights. Thus, the linear programming can be solved by the Lingo software. With the weight solved by the above linear programming, we can compute the net flow of the alternatives and rank the alternative from best to the worst one.

IV. A NUMERICAL EXAMPLE

Considering that the alternative set had seven alternatives, and judgment criteria set had five criteria. The following table illustrates the evaluations of the alternatives on the judgment criteria.

TABLE 1 EVALUATION OF ALTERNATIVE ON EACH CRITERION

	criterion1	criterion2	criterion 3	criterion 4	criterion 5
Alternative 1	30	6	5	3.5	18
Alternative 2	70	3	8	4.5	24
Alternative 3	50	1	4	5.5	15
Alternative 4	100	4	6	8.0	20
Alternative 5	60	2	8	7.5	16
Alternative 6	80	5	5	4.0	21
Alternative 7	45	4	7	6.5	19

The following table illustrates the preference and indifference threshold.

TABLE II THRESHOLD OF PREFERENCE AND INDIFFERENCE

	criterion 1	criterion 2	criterion 3	criterion 4	criterion 5
q	15	1	1	1.5	3
p	25	2	2	2.5	6

The decision maker gave the preference relation of the alternatives: $a_4 \succ a_2 \succ a_1 \succ a_3$.

The weights of criteria can be derived by using the above linear programming: $\omega_1 = 0.316$， $\omega_2 = 0.302$， $\omega_3 = 0.128$， $\omega_4 = 0.142$ $\omega_5 = 0.112$。

At last, the net flow of the alternative can be induced.

TABLE III NET FLOW OF ALTERNATIVE

Alternative	1	2	3	4	5	6	7
Net flow	0.51	0.547	0.432	0.713	0.467	0.685	0.601

Thus, the preference relation of alternatives $a_4 \succ a_6 \succ a_7 \succ a_2 \succ a_1 \succ a_5 \succ a_3$ is established.

CONCLUSION

In this paper, a method of inferring the weight of multi-criteria decision making problem is proposed. A linear program is constructed with the PROMETHEE method. At last, a numerical experiment is conducted to illustrate the above method.

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